Capon Beamspace Beamforming for Distributed Acoustic Arrays

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ABSTRACT

This paper presents the advantages of using distributed arrays with the beamspace implementation of a wideband Capon algorithm. Distributed arrays have recently demonstrated a decreased sensitivity to environmental coherence loss and array mismatches. In an attempt to build upon this robustness, beamspace preprocessing is applied in this paper. Beamspace allows sector-focused beamforming and reduced computational complexity. A unique property of this implementation of beamspace, namely being indifferent to losses of entire channels of data, has been demonstrated. This implementation of beamspace to Capon beamforming does not require beam orthogonalization, thus saving computational time, especially in whitening process.

Keywords: D istributed Acoustic Sensors, Wideband DOA Estimation, Capon Beamforming, Beamspace Preprocessing

1. INTRODUCTION

Unattended passive acoustic sensors are becoming increasingly important in a variety of military surveillance applications. These sensors are rugged and dependable and are capable of capturing the temporally and spectrally overlapping sources. It has also recently been shown[1] that randomly distributed acoustic sensor arrays, because of their lack of a sidelobe structure, are optimal choices when trying to localize scattered or nearfield sources, or if there are array manifold mismatches, sensor location uncertainties, or other coherence losses due to the environment. The distributed configuration also provides a narrow main beam with which to localize tightly grouped sources. Effective source localization with a distributed array requires adequate spatial diversity, which is dependent on the spectral content of the source, its distance from the array, and other factors. This reduces the sidelobe structure and guarantees more robust performance to the above mentioned problems. The robustness claimed in this paper does not refer to the robustness to perturbations in the signal model or steering vector as in,[2] rather it is robustness to channel data loss.

In this paper, wideband beamforming methods were applied to distributed array configurations, beamspace preprocessing has promise due to its robustness to error in the specific practical scenario of channel data loss. Beamspace beamforming has been extensively studied and applied to MUSIC, Root-MUSIC, ESPRIT, and other high-resolution direction of arrival (DOA) estimation methods[3] and used on various array geometries. This paper studies the application of beamspace processing and its performance when used in conjunction with the wideband Capon algorithm on the distributed sensor array. Interesting new properties of the distributed acoustic sensor array are unveiled when using beamspace preprocessing algorithm. These properties are studied in this paper both experimentally and theoretically.

The organization of this paper is as follows. The signal model is introduced in Section 2 along with the wideband Capon beamformer. The extension of wideband Capon beamforming using beamspace preprocessing is then described in Section 3. Properties resulting from the application of the wideband beamspace Capon method are discussed in Section 4. Concluding remarks and observations are given in Section 5.

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2. WIDEBAND SIGNAL MODEL AND CAPON BEAMFORMING

Let us consider an array of M sensors that receive the wavefield generated by d wideband sources in the presence of an arbitrary noise wavefield. The array geometry can be arbitrary but known to the processor. The source signal vector $\mathbf{s}(t) = [s_1(t), s_2(t), \dots s_d(t)]^T$ is assumed to be zero mean and stationary over the observation interval T_0. The source spectral density matrix is denoted by $P_{\mathbf{s}}(f)$, $f \in [f_c - BW/2, f_c + BW/2]$ with the bandwidth BW comparable to the center frequency f_c. The spectral density matrix $P_{\mathbf{s}}(f)$ is a $d \times d$ nonnegative Hermitian matrix, unknown to the processor. The noise wavefield is assumed to be independent of the source signals with $M \times M$ noise spectral density matrix $P_{\mathbf{n}}(f)$. Then, the spectral density matrix of the M-dimensional array output $\mathbf{x}(t)$ may be written as

$$P_{\mathbf{x}}(f) = A(f, \boldsymbol{\phi}) P_{\mathbf{s}}(f) A^H(f, \boldsymbol{\phi}) + P_{\mathbf{n}}(f) \tag{1}$$

where $A(f, \boldsymbol{\phi}) = [\mathbf{a}(f, \phi_1), \mathbf{a}(f, \phi_2), \dots, \mathbf{a}(f, \phi_d)]$ is the $M \times d$ array manifold matrix of the sensor array, with respect to some chosen reference point, $\mathbf{a}(f, \phi_i)$ is the steering vector of the i^{th} source, and $\boldsymbol{\phi} = [\phi_1, \dots, \phi_d]$ is the bearing angle vector of the sources. It is assumed that $M > d$ and that the rank of $A(f, \boldsymbol{\phi})$ is equal to d at any frequency and angle of arrival. Taking the DFT on the output vector, $\mathbf{x}(t)$, from non-overlapping segments of the times series of length ΔT composed of K averaged snapshots we obtain a frequency decomposition of the output. We denote the j^{th} narrowband component of all the DFT outputs of the k^{th} snapshots by vector $\mathbf{x}_k(f_j)$, $k = 1, 2, \dots, K$, and $j = 1, 2, \dots, J$. Then the spatial covariance matrix of the recorded data for the j^{th} narrowband component $\mathbf{x}_k(f_j)$ is

$$\begin{aligned} \mathrm{R}_{\mathbf{xx}}(f_j) \;\approx\; & \mathrm{P}_{\mathbf{x}}(f_j) = \mathrm{A}(f_j, \boldsymbol{\phi}) \mathrm{P}_{\mathbf{s}}(f_j) \mathrm{A}^H(f_j, \boldsymbol{\phi}) \\ & + \mathrm{P}_{\mathbf{n}}(f_j), \quad j = 1, 2, \dots, J. \end{aligned} \tag{2}$$

where ΔT is assumed to be $\Delta T = 1$.

In[4] , several wideband Capon DOA estimation methods were developed that use different spectral averaging schemes to estimate ϕ_i's using the spatial covariance matrix, $R_{\mathbf{xx}}$, of the data. At frequency bin f_j, the rank-1 narrowband Capon beamformer $\mathbf{w}(f_j, \theta)$ is defined as the vector that maximizes the signal-to-noise/interference at the output of the beamformer, $\mathbf{z}_k(f_j) = \mathbf{w}^H(f_j, \theta)\mathbf{x}_k(f_j)$, for the scalar look-direction angle, θ. This problem can be posed[3] as a minimization problem which leads to the narrowband rank-1 Capon beamformer

$$\mathbf{w}(f_j, \theta) = \frac{\mathrm{R}_{\mathbf{xx}}^{-1}(f_j)\mathbf{a}(f_j, \theta)}{\mathbf{a}^H(f_j, \theta)\mathrm{R}_{\mathbf{xx}}^{-1}(f_j, \theta)\mathbf{a}(f_j, \theta)} \tag{3}$$

Where θ is the search angle. The narrowband power spectrum becomes

$$q(f_j, \theta) = \frac{1}{\mathbf{a}^H(f_j, \theta)\mathrm{R}_{\mathbf{xx}}^{-1}(f_j)\mathbf{a}(f_j, \theta)} \tag{4}$$

Note $\mathrm{R}_{\mathbf{xx}}(f_j) = \sum_{k=1}^{K} \mathbf{x}_k(f_j)\mathbf{x}_k^H(f_j)$ is the sample covariance matrix and it is assumed that $\mathbf{w}(f_j, \theta)$ is independent of k within one snapshot. The geometrically averaged wideband Capon has the power spectrum obtained from the mean of the spectra at different frequencies given by

$$Q_G(\theta) = \prod_{j=1}^{J} q(f_j, \theta) = \prod_{j=1}^{J} \frac{1}{\mathbf{a}^H(f_j, \theta)\mathrm{R}_{\mathbf{xx}}^{-1}(f_j)\mathbf{a}(f_j, \theta)} \tag{5}$$

which demonstrated the best overall results among all the possible spectral averaging methods in.[4] This wideband extension aids in localizing multi-spectral sources. These results are also consistent with the findings in[5] and[6] concerning optimal (in the ML sense) wideband beamforming leading to a geometric mean beamformer.

From the power spectrum the DOA's are then estimated by searching for the θ of the peaks of the Capon spectrum $Q_G(\theta)$; a pre-specified number of peaks are chosen based on whether or not they reach a threshold which is dependent on the value of the largest peak. It should be noted that this does not infer the SNR or the actual noise power, it only picks the maximum power estimates.

3. WIDEBAND BEAMSPACE METHOD

The beamspace method reduces the complexity and degrees of interference cancellation freedom in an array by steering a group of *beams* instead of the phasings of the individual sensors. The $M \times M_{bs}$ beamspace matrix B_{bs} is a projection matrix from the element space (size $\mathbb{C}^M$) to the beamspace (size $\mathbb{C}^{M_{bs}}$)($M \geq M_{bs}$). Finding covariance with respect to these beams instead of the elements focuses the region of interest (or region of active interference cancellation) and dramatically attenuates signals outside this region. Beamspace preprocessing projects the input data vector into a subspace where the signal of interest will be extracted easier. It should be noted that this produces ambiguities because the projected space is smaller than the element space. Hence, the algorithm acts as a spatial bandpass filter. This also reduces the amount of processing as the size of the covariance matrix reduces from $M \times M$ to $M_{bs} \times M_{bs}$. This could be especially important in medium to large distributed sensor networks.

It is important that the beams be orthogonal, this ensures that any signal arriving along the main lobe of a particular beam will not produce output from any other beam. The $M \times M_{bs}$ matrix B_{bs}, ($M_{bs} = M$ for full-dimension), is formed with M_{bs} standard steering vectors of each beam. These beams must be steered through all electrical angles to obtain the power spectrum. A general form of an non-orthogonalized beamspace matrix is

$$\begin{aligned} B_{no}(f_j, \theta) &= [\mathbf{b}(f_j, \delta_{-P} + \theta) \ldots \mathbf{b}(f_j, \delta_0 + \theta) \\ &\quad \ldots \ \mathbf{b}(f_j, \delta_P + \theta)] \end{aligned} \tag{6}$$

where

$$\mathbf{b}(f_j, \tau) = \begin{bmatrix} e^{j2\pi f_j/c(\alpha_0 \cos(\tau) + \beta_0 \sin(\tau))} \\ e^{j2\pi f_j/c(\alpha_1 \cos(\tau) + \beta_1 \sin(\tau))} \\ \vdots \\ e^{j2\pi f_j/c(\alpha_{M-1} cos(\tau) + \beta_{M-1} sin(\tau))} \end{bmatrix}, \tag{7}$$

and $\delta_{-P}, ..., \delta_0, ..., \delta_P$ are the angles of each of the beams, $\mathbf{b}(f_j, \tau)$, in the beamspace matrix, c is the speed of sound in air, α_i and β_i are the horizontal and vertical positions of the i^{th} sensor relative to the reference position. To ensure orthogonality of the beamspace matrix, we compute

$$B_{bs} = B_{no}[B_{no}^H B_{no}]^{-\frac{1}{2}}. \tag{8}$$

Clearly orthogonality results in $B_{bs}^H B_{bs} = I_{M_{bs}}$. The received data is then preprocessed using the beamspace matrix B_{bs} prior to any beamforming process. The processing steps for the wideband beamspace method (although not novel) are listed below.

1. Transform the array output, $\mathbf{x}_k(f_j)$, $k = 1, 2, ..., K$ using

$$\mathbf{v}_k(f_j) = B_{bs}^H(f_j, \theta)\mathbf{x}_k(f_j) \tag{9}$$

   where $B_{bs}(f_j, \theta)$ is the $M \times M_{bs}$ beamspace matrix whose columns are the orthogonal beampatterns (or other optimized beams) centered around θ and frequency f_j. The resulting sample covariance matrix of the transformed array output, $\mathbf{v}_k(f_j)$ is

$$\begin{aligned} R_{\mathbf{vv}}(f_j) &= B_{bs}^H(f_j, \theta) R_{\mathbf{xx}}(f_j) B_{bs}(f_j, \theta) \\ &\approx B_{bs}^H(f_j, \theta) A(f_j, \theta) \mathrm{P}_{\mathbf{s}}(f_j) A^H(f_j, \theta) B_{bs}(f_j, \theta) \\ &\quad + B_{bs}^H(f_j, \theta) \mathrm{P}_{\mathbf{n}}(f_j) B_{bs}(f_j, \theta). \end{aligned} \tag{10}$$

2. The transformed steering vector is given as

$$\mathbf{a}_{bs}(f_j, \theta) = B_{bs}^H(f_j, \theta)\mathbf{a}(f_j, \theta) \tag{11}$$

   and the beamformer output becomes $\mathbf{z}_k(f_j, \theta) = \mathbf{w}_{bs}^H(f_j, \theta)\mathbf{v}_k(f_j)$, where

$$\mathbf{w}_{bs}(f_j, \theta) = \frac{R_{\mathbf{vv}}^{-1}(f_j)\mathbf{a}_{bs}(f_j, \theta)}{\mathbf{a}_{bs}^H(f_j, \theta) R_{\mathbf{vv}}^{-1}(f_j) \mathbf{a}_{bs}(f_j, \theta)}. \tag{12}$$

3. The wideband geometric mean beamspace power spectrum output then becomes

$$Q_{G_{bs}}(\theta) = \prod_{j=1}^{J} \frac{1}{\mathbf{a}_{bs}^H(f_j,\theta) R_{\mathbf{vv}}^{-1}(f_j) \mathbf{a}_{bs}(f_j,\theta)}. \tag{13}$$

The beamspace preprocessing method requires more computations when compared to the standard Capon. Mostly due to the whitening process for the beamspace matrix at each look direction and frequency bin. However, in more general applications, especially those involving large numbers of sensors, the beamspace algorithm decreases the overall computational complexity. Beamspace is also very effective when the normalized average period (K/M) is small. The algorithm allows the statistical convergence of the covariance matrix to steady state because of its reduced size.[3]

An important feature of the Capon Beamspace method is that the whitening that is typically done on the beamspace beams to ensure orthogonality, is not required in the Capon extended method. This is shown below.

Let us consider the spectrum of narrowband beamspace Capon beamformer at frequency f_j, i.e.

$$q_{bs}(f_j,\theta) = \frac{1}{\mathbf{a}_{bs}^H(f_j,\theta) \mathrm{R}_{\mathbf{vv}}^{-1}(f_j) \mathbf{a}_{bs}(f_j,\theta)} \tag{14}$$

where $\mathrm{R}_{\mathbf{vv}}^{-1}(f_j)$ and $\mathbf{a}_{bs}^H(f_j,\theta)$ are defined in (3), respectively in terms of matrix $B_{bs}(f_j,\theta)$. Using these expressions and the whitening equation, (8), we can easily show that

$$q_{bs}(f_j,\theta) = \frac{1}{a^H(f_j,\theta) B_{no}(f_j,\theta) \left(B_{no}(f_j,\theta)^H \mathrm{R}_{\mathbf{xx}}^{-1}(f_j) B_{no}(f_j,\theta)\right)^{-1} B_{no}^H(f_j,\theta) a(f_j,\theta)} \tag{15}$$

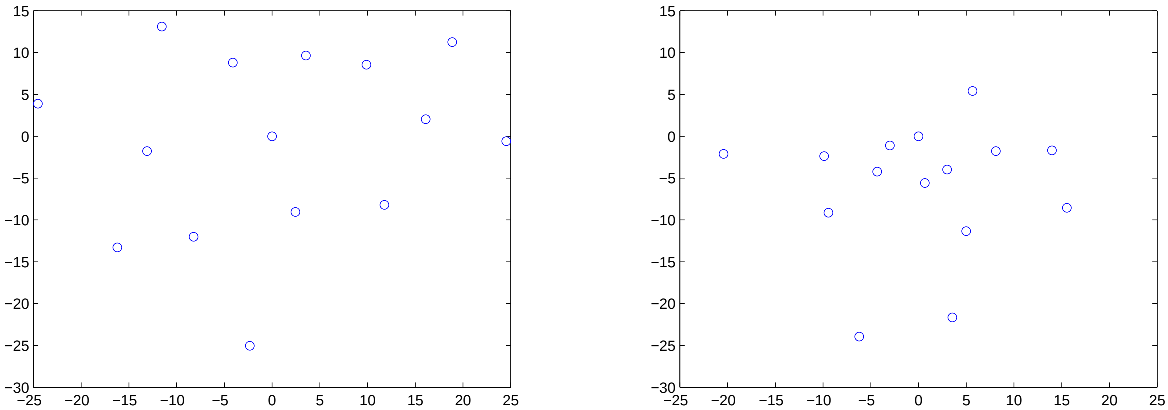
i.e. the beamspace Capon without orthogonalization.

This shows that orthogonalization of the beamspace matrix for use in the Capon algorithm is not necessary, thus saving a $\mathcal{O}(N^3)$ computations per look angle and frequency bin.

4. PROPERTY OF BEAMSPACE PROCESSING IN DISTRIBUTED SPARSE ARRAYS: ROBUSTNESS TO CHANNEL DATA LOSS

Several field testing exercises have recently carried out using Telos-B mote-based (ZigBee compliant) sensor nodes forming various random distributed wireless sensor networks. A proprietary data synchronization and transmission routine was used to sychronize the data of all the nodes. These field tests were carried out at Colorado State University (CSU) football stadium parking lot in Fort Collins, CO on April 8 and 12, 2006. The data sets were collected using two wireless sensor network configurations employing fifteen randomly distributed mote-based nodes for both single and two vehicles runs which used light wheeled U-Haul (17ft) and Ryder (16ft) diesel trucks. The random distributed sensor array configurations not only offer robustness to environmental and sensor position errors, but also to sensor failure or data loss especially when used with beamspace beamforming. The data is collected using a network of wireless acoustic sensors. All 15 nodes transmit the streaming of synchronized recorded time series data in a single-hop to a base station (laptop). The sensor locations were surveyed using a relatively accurate GPS within an average error radius of 0.1m. The worst case localization error radius was around 0.25m for some sensors. The data was sampled at $f_s = 1024$Hz. Two unique random sensor deployment configurations within an area of radius 25m were considered. Figures 1(a) and (b) show these two random deployment layouts. Configuration II offers more spatial and frequency diversity and hence provides better high to low frequency analysis. Configuration I, however, offers better separation of close vehicles due to more uniform spreading of the sensors. The data of two experiments were used not only to show the real potential of the distributed sensors for acoustic tracking of vehicles but also to evaluate the empirical performance of different algorithms presented in this paper.

In the system used for collecting the given data, sensor failure or packet losses happen often and are almost unavoidable. Closer examination, however, revealed that in this case the mote that failed to collect useful data had a bad microphone or amplifier circuit. In case of failure, one or more sensors nodes do not collect data. To account for the failure in the DOA estimation process, one has to manually remove the failed data channels before



(a) Configuration I

(b) Configuration II

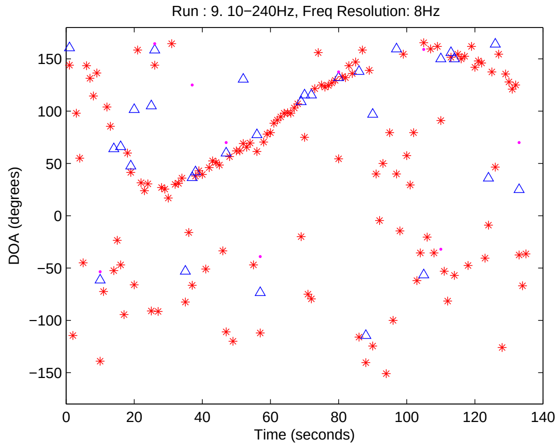
Figure 1. Two randomly distributed sensor configurations using 15 mote-based sensors.

beamforming. Removing sensor data manually in a real-time implementation of a distributed sensor network is not practical, especially in battlefield scenarios. To demonstrate the usefulness of the wideband beamspace method developed in Section 3 in these situations, this method is applied without using the knowledge of the failed sensor. That is, the bad data of this sensor was included in the wideband Capon beamspace beamforming process. The beamspace preprocessing allows the beamforming to continue and provide excellent results even in the presence of multiple failed data channels. Additionally, the beamspace Capon method also proves to be an excellent algorithm when a high level of wind noise is present in the collected data.

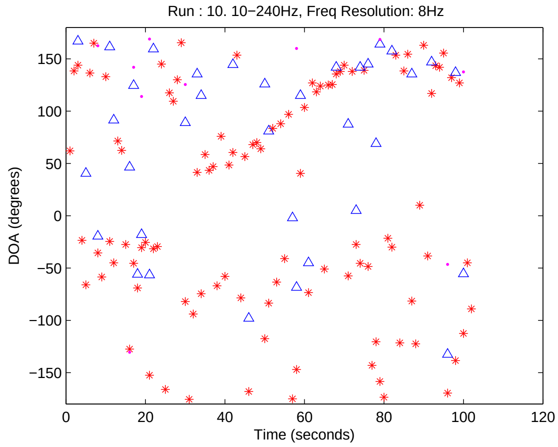
Figures 2 (a)-(b) show the results for two different runs with a failed sensor node using the standard wideband Capon beamforming. The DOA track shows the vehicle movement pattern that varies significantly snapshot-to-snapshot. These results, before beamspace preprocessing, also exhibit a large number of erroneous DOA estimates. Figures 2 (c) and (d) show the effect of manually removing the failed data channels and then applying the wideband Capon. As can be seen, this removal indeed restores the peak estimates of beamformed spectra and provides reasonably clear vehicle movement pattern. Figures 2 (e) and (f) show the results when using the beamspace preprocessing and the wideband Capon beamforming. It is evident that the beamspace preprocessing led to significant improvement in the quality of the estimates and DOA resolution without the need to identify and remove the failed channel. Additionally, the DOA estimates generated using the beamspace processing exhibit a greatly reduced variance between the estimates, forming a much smoother movement track pattern.

Figures 3(a)-(c) show the DOA estimates for another case where two sensor nodes failed simultaneously. Similar observations can also be made in this case, i.e. the improvements of the beamspace preprocessing over the standard wideband Capon method with or without bad data removed. Additionally, there appears to be another point even in the beamspace preprocessed data where the peak estimates do not accurately illustrate the presence of the vehicle. There is a sudden loss in good estimated peaks and this is believed to be related to the number of channel failures and the axis along which these failed nodes exist. Therefore, when a source direction is perpendicular to this axis, the bad data is more harmful to the beamforming process and seems to even affect the performance of the beamspace preprocessing algorithm.

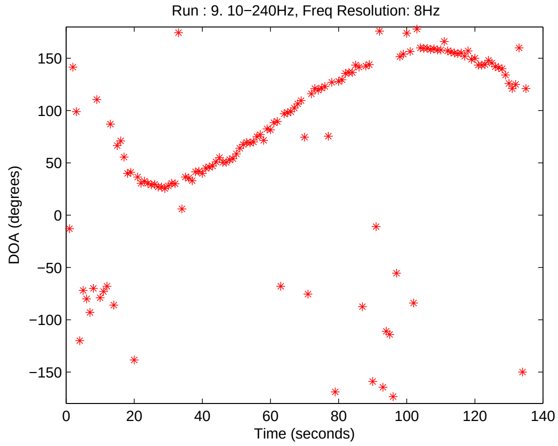
An explanation of the beamspace robustness to failed sensor nodes is that this algorithm performs a complex weighting the sensor element inputs in such a way that it only relies on a subset of the sensors in the array. This would not normally prevent errors in the beamforming process on a structured array. But in this case, because of the random distributed configuration and the angular focusing property of the beamspace method, the input from the failed sensor adds only small errors into the steering vector. This is in contrast to the standard Capon, which attempts to perform interference cancellation over the entire angular field of view, and hence, receives the full effect of the mismatched phases from the failed channels. Thus, because of the structure and the beamspace method's insensitivity to low SNR, accurate DOA estimates can still be obtained.



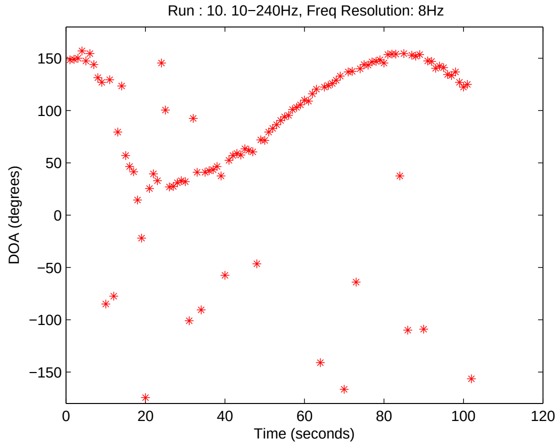
(a) Standard Capon for first run



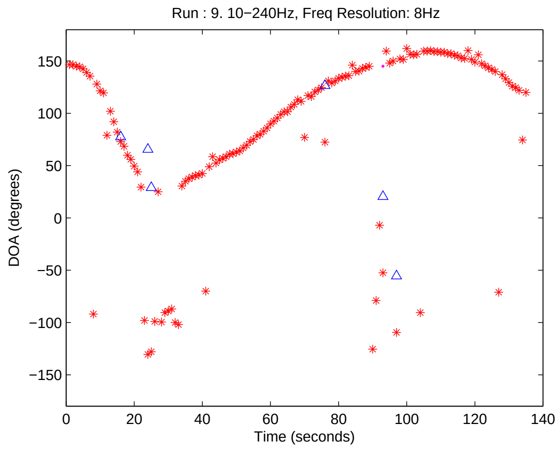
(b) Standard Capon for second run



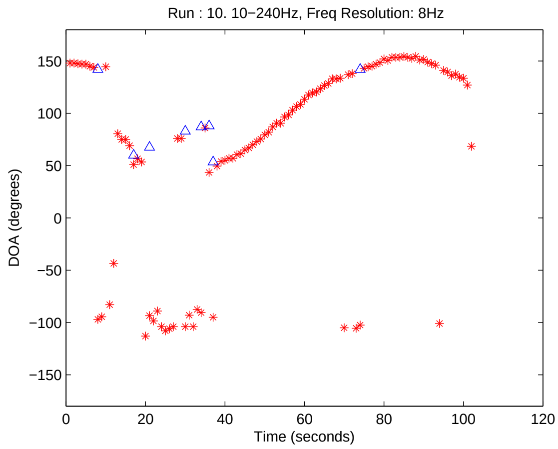
Standard Capon with failed node removed for first run



(d) Standard Capon with failed node removed for second run

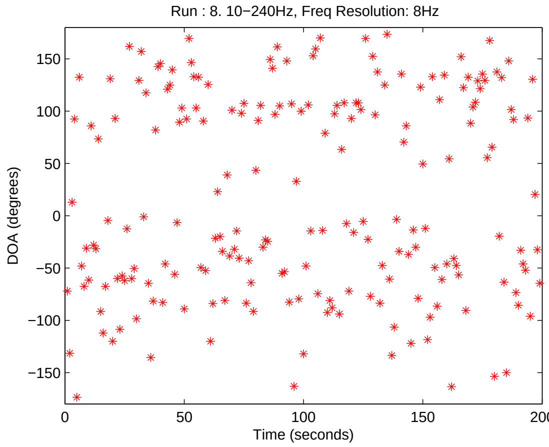


(e) Unwhitened beamspace Capon for first run

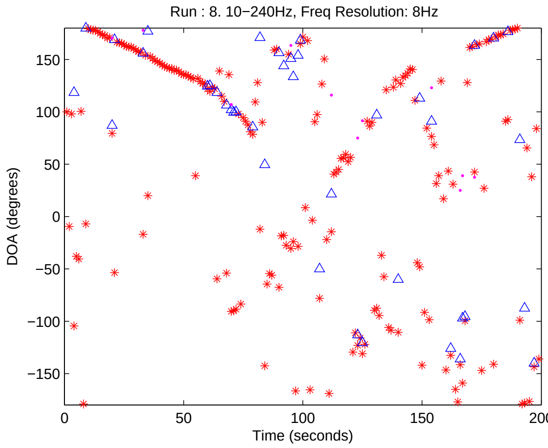


(f) Unwhitened beamspace Capon for second run

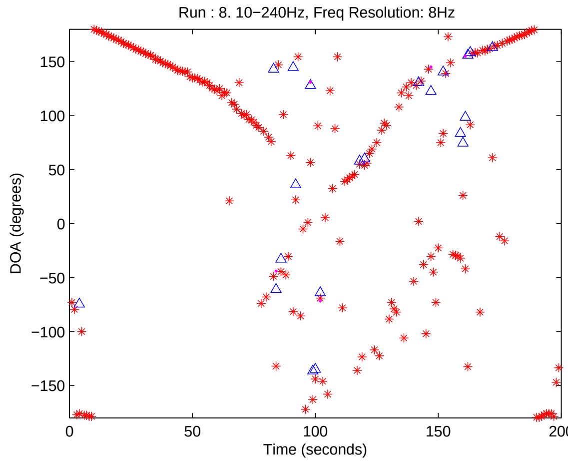
Figure 2. Comparison of standard wideband Capon and wideband Capon with beamspace preprocessing in the presence of a failed sensor node using array configuration II. The stars mark the first estimate (largest peak), the triangles represent second peak estimates.



(a) Standard Capon for third run



(b) Unwhitened beamspace Capon for third run



(c) Standard Capon with failed nodes removed for third run

Figure 3. Comparison of standard wideband Capon and wideband Capon with beamspace preprocessing in the presence of two failed sensor nodes using array configuration I. The stars mark the first estimate (largest peak), the triangles represent second peak estimates.

5. CONCLUSIONS

This paper considers the problem of acoustic source localization in distributed sensor networks using high-resolution beamforming and beamspace preprocessing. It has been demonstrated with real data sets that the distributed sensor network, when a spatial filtering algorithm is used, i.e. beamspace preprocessing, is very robust to channel failure, as well as being robust to source and environmental mismatches, and sensor position errors. The beamspace method also reduces computational complexity of the overall algorithm. Actual beamformed results are presented to confirm the robustness of the proposed algorithm for two different distributed sensor configurations consisting of 15 sensor nodes (motes). These results show better peak estimation for forming vehicle movement patterns by using beamspace preprocessing without removing the failed data channels.

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